Downloaded from rspa.royalsocietypublishing.org

PROCEEDINGS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

A New Current Weigher, and a Determination of the Electromotive Force of the Normal Weston Cadmium Cell

W. E. Ayrton, T. Mather and F. E. Smith

Proc. R. Soc. Lond. A 1907 **80**, 12-18
doi: 10.1098/rspa.1907.0068

Email alerting service

Receive free email alerts when new articles cite this article - sign up in the box at the top right-hand corner of the article or click **here**

To subscribe to *Proc. R. Soc. Lond. A* go to: **http://rspa.royalsocietypublishing.org/subscriptions**

This journal is © 1907 The Royal Society

Downloaded from rspa.royalsocietypublishing.org

annealing respectively are of a different order to those which are due to allotropic or dimorphic transformations, or to the separation or rearrangement of constituents. The results here recorded lead us to believe that mechanical work produces only internal rearrangement of the metallic grains or molecules, but does not lead to any chemical or physical changes such as are correctly regarded as allotropic. We have met with no evidence to support the view that allotropic change results from mechanical work.

A New Current Weigher, and a Determination of the Electromotive Force of the Normal Weston Cadmium Cell.

By Professor W. E. Ayrton, F.R.S., and T. Mather, F.R.S., Central Technical College, London, and F. E. Smith, A.R.C.S., National Physical Laboratory, Teddington.

(Received June 5,—Read June 27, 1907.)

(Abstract.)

Introductory.

The instrument described is the outcome of conversations between the late Professor J. Viriamu Jones, F.R.S., and one of the authors (W. E. A.), on their return from the British Association Meeting held in Toronto in 1897. Its object was to determine "the ampere" as defined in the C.G.S. system, to an accuracy comparable with that attained in the absolute determination of the ohm by Lorenz's apparatus, an account of which was given by Professors Ayrton and Jones at the Toronto Meeting.*

Professor Jones had previously developed a convenient formula for calculating the electromagnetic force between a helical current and a coaxial current sheet, viz.,

$$F = \gamma_h \gamma (M_2 - M_1),\dagger$$

where γ_h is the current in the helix, γ the current per unit length of the current sheet, and M_1, M_2 the coefficients of mutual induction of the helix and the two ends of the current sheet respectively. By using coaxial coils with single layers of wire wound in screw-thread grooves, advantage could be taken of the above formula.

* See 'B. A. Report,' Toronto, 1897, p. 212.

† 'Roy. Soc. Proc.,' vol. 63, p. 204.

The Royal Society is collaborating with JSTOR to digitize, preserve, and extend access to *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character.* www.jstor.org

Downloaded from rspa.royalsocietypublishing.org

A preliminary apparatus was made at the Central Technical College in 1897–8, and used to obtain experience as to the conditions necessary for successful operation.* The design of the proposed instrument was then proceeded with.

By employing the arrangement devised by one of the authors (W. E. A.), of having double-threaded screw grooves wound with separate bare wires subsequently connected in series, it was possible to prevent any uncertainty existing regarding leakage between adjacent turns of a spiral. At the same time, the absence of any silk covering on the wires wound on bare marble cylinders enabled very great precision in the measurement of the dimensions of each coil to be attained.

Complete working drawings and specifications of the proposed instrument and its adjustable support were prepared at the Central Technical College in 1898–99, Mr. J. P. Gregory, a then student of the College, rendering very valuable assistance in this work. Grants amounting to £300, for the construction of the balance, were made by the British Association for the Advancement of Science in 1897 and 1898, and Sir Andrew Noble, F.R.S., kindly presented the adjustable phosphor bronze stand, designed to support the instrument. The physical balance was built by Mr. L. Oertling, of London; and the electrical portions were made at the National Physical Laboratory under the supervision of the Director, Dr. R. T. Glazebrook, F.R.S.

General Description.

The instrument consists of a very sensitive physical balance, with a 20-inch beam,† supporting from each end a coil with vertical axis; these coils hang coaxially within fixed coils carried from the base. All the coils are wound in single layers on hollow marble cylinders having *double-threaded* screw grooves cut on their surfaces, the fixed cylinders being about 13 inches diameter, 11 inches high, and 2 inches thick, and the suspended cylinders $8 \times 6 \times \frac{1}{2}$ inches. Each fixed cylinder carries two windings, an upper and a lower, in which the current circulates in opposite directions, so that one repels and the other attracts, the current carrying winding on the suspended cylinder hanging symmetrically between them. Each winding, on both fixed and suspended cylinders, consists of two helices occupying adjacent grooves of the double-threaded screws; there are thus 12 wires in all on the four cylinders, and these are connected all in series in the ordinary use of the balance.

Connection with the suspended coils is made by flexible silver wires 1 mil

* 'B. A. Report,' Bristol, 1898, p. 157; also 'Journ. Inst. Elec. Engrs.,' vol. 35, p. 12.

† The balance carries about $5\frac{1}{2}$ kilos, and turns with 1/10 milligramme.

Downloaded from rspa.royalsocietypublishing.org

in diameter, arranged 80 in parallel, and all other leads to and from the coils are of small concentric cable. These cables run to a plug board and commutators, whereby the current in any or all of the 12 helices may be reversed. Adjacent helices can also be disconnected from each other, and electrically grouped so that the insulation between all the pairs may be found by a single test.

When in use for measuring current, the directions of circulation in the several coils are arranged so that they all produce a torque in the same sense on the balance beam, and this torque is balanced by suitable masses placed on the scale pans. Reversal of the current, in the fixed coils only, produces an apparent change of weight, which is a measure of the square of the current employed. The position of the beam of the balance is observed by viewing through a microscope a finely divided scale carried by the pointer.

The masses used to balance the electromagnetic forces can be manipulated quickly and conveniently without opening the case of the instrument, four weight lifters being provided for this purpose. To prevent change of level of the suspended cylinders on removing or replacing the weights, the scale pans are carried on separate planes resting on the knife edges from which the suspended coils hang.

For convenience in erection and adjustment the fixed coils are supported on phosphor bronze slide rests, capable of about half an inch movement in two horizontal directions and of 14 inches vertical displacement.

Advantages of Duplication of Coils.

The arrangement of having a set of coils at each end of the balance beam has several advantages. Independent determinations may be made on the two sets, or, by using both together, the forces are approximately doubled. Differential tests can also be made so that one set serves as a check on the other. The chief advantage, however, is the symmetry obtained, which neutralises to a very great extent the disturbances arising from convection currents of air and change of air buoyancy.

The double winding is also of great utility in this respect, as it permits of an electromagnetic weighing being made with one set of coils without introducing the error, due to drift of zero, which would occur if that set alone were heated by the passage of the current. This is effected by arranging the connections so that the currents in adjacent helices on one pair of cylinders are in opposite directions; they have then no electromagnetic effect, and yet are heated by the passage of the current just as much as the coils on the other pair.

Downloaded from rspa.royalsocietypublishing.org

Magnetic Tests.

Before constructing the parts of the balance, magnetic tests were made on all the materials intended to be used, and in cases where the permeability differed appreciably from unity those parts were rejected.

When the instrument was completed it was used to test itself, but no trace whatever of magnetism could be detected.

Construction and Winding.

The material employed for the fixed and suspended cylinders is Carrara Statuary Marble, which, after being turned approximately to size, was baked in an oven at 140° C. for 30 hours and then immersed in hot paraffin wax. The turning was then completed, the **V** grooves cut and bare copper wire, No. 24 S.W.G., wound in them under tension, numerous measurements of the diameter of the wire being made during the winding. Special arrangements were made to ensure that each helix contained an exact number of turns, and that the two helices of one pair start and end in the same diametral plane.

The pitch of the screw grooves is 1/18 inch: each helix on a suspended cylinder has 92 turns, making 184 in all, and those on each fixed cylinder 90 turns, making a total of 360. These turns on the fixed cylinders are in two portions, upper and lower, with an unwound space of 2/18 inch between them. The axial length of the suspended coil is, therefore, equal to the distance between the central planes of the upper and lower windings on the fixed cylinder. On all the cylinders the total number of turns is 1088 and the total length of wire about 980 metres.

Measurement of Coils.

Measurements of the axial lengths of the windings were made by a cathetometer, and of the diameters by a special measuring machine obtained from Messrs. Stanley, of London. An optical lever used with the latter instrument gave a deflexion of 2 mm. for a movement of the micrometer of 1 μ (1/1000 of a millimetre), so the dimensions could be determined with great precision. Such accuracy in the measurement of the diameters was rendered possible owing to the use of bare wire wound on bare marble.

About 120 diameters were measured on each suspended cylinder, and about 220 on each fixed one, the probable error of the mean diameter of any cylinder amounting to about five in a million. All the cylinders are very nearly perfect, the ellipticities and the conicalities being extremely small.

Downloaded from rspa.royalsocietypublishing.org

After measuring the coils and insulating adjacent helices, the windings were coated with melted wax, and again measured in numerous places from which the wax was temporarily removed; no appreciable change in dimensions could be found.

Erection and Adjustment.

To facilitate the setting of the cylinders, two spirit-levels are mounted on the upper plane end of each, and adjusted so that when these indicate level the axis of the corresponding cylinder is truly vertical. Mechanical indicators are also provided for showing when the cylinders are co-axial and when the middle planes of the windings on the fixed and suspended cylinders coincide. The vertical and horizontal adjustments can also be tested by electrical methods which are simple and very accurate; these were adopted in the final settings, the error introduced by faulty adjustment amounting to something less than 1 part in 5 millions.

Calculation of Mutual Inductions and Forces between the Coils.

The formulæ employed are given in Professor J. V. Jones' paper, mentioned on p. 12, and were employed by two of the authors (T. M. and F. E. S.) to calculate *independently* the mutual inductions and the forces between the fixed and suspended coils when a current of 1 ampere circulates in them. The values of $M_2 - M_1$, defined on p. 12, calculated (a) with logarithms and (b) by calculating machine, were in very close agreement, the difference being less than 1 in a million. The actual numbers are 25962·02 cm. and 25962·04 cm. respectively for the left-hand set of coils, and 25960·45 cm. and 25960·43 cm. for the right-hand set. This order of accuracy in the values of the coefficients of mutual induction is rendered quite real by a knowledge of the exact position of the bare conductors and the certainty that no current leaks from one convolution of wire to any other. In both cases the sum for both sets of coils is 51922·47 cm., and this number was used in most of the determinations.

Taking the value of g, the acceleration of gravity at Bushy, as 981·20, and the length of winding on the suspended cylinders as 12·9830 cm., the change of apparent mass on reversal of 1 ampere in both sets of coils becomes

$$m = 14{\cdot}9992_8 \text{ grammes.}$$

This assumes that no forces exist between the fixed coils on one side of the balance and the suspended ones on the other side. As a matter of fact, such forces are present, but can be eliminated by taking two sets of readings, one set in which the cross actions assist the direct ones and the other set in which

Downloaded from rspa.royalsocietypublishing.org

these forces oppose each other. These sets give what we have called (D + S) (direct + secondary) and (D − S) observations respectively. The change from one condition to the other is effected by simply reversing the direction of current in all the coils on one side of the balance; this leaves the direct forces unchanged in direction but reverses the secondary forces.

In the majority of the determinations (D + S) and (D − S) observations were taken in succession and the current calculated from the expression

$$\text{amperes} = \sqrt{(m'/29{\cdot}99856)},$$

where m' is the sum of the balancing masses in the (D + S) and (D − S) tests.

Tests on Cadmium Cells.

To determine the E.M.F. of a cell, an electric current supplied by a 110-volt storage battery was passed through a standard resistance of approximately 1 ohm, in series with the current weigher, and adjusted in strength until the P.D. between the terminals of the resistance balanced the E.M.F. of the cell. The magnitude of this current was then determined by the instrument, the cell and the resistance being kept at very nearly constant temperature during the measurements. Switches with copper contacts and terminals, to minimise thermal E.M.F.'s, were used in the cell circuit, and reversals of current and of the cell were sometimes made to eliminate inaccuracies which might otherwise be introduced by such E.M.F.'s.

As the steadiness of the balance was much greater when the instrument was cold than when heated by the long continued flow of the current in the coils, our usual procedure was to make one complete set of observations, (D + S) and (D − S), in the morning and another in the afternoon. The interval between the morning and afternoon readings was often devoted to silver deposit determinations, the combination of cell and standard resistance being used as a secondary standard of current during the depositions. An account of work on silver is well advanced, and will be published shortly.

In all some 71 observations have been made on a certain cadmium cell (N.P.L., No. 2), using both sets of coils on the balance, and 13 observations in which one or other of the two sets was employed. The agreement between the individual results obtained with the two sets of coils is remarkable, the average difference from the mean amounting to only six parts in a million. The whole series of observations extended over a period of 19 months (September, 1905, to April, 1907), and during that interval the coils of the balance were reset five times. No determination made has been omitted, except those in which the observations were of such a nature that

Downloaded from rspa.royalsocietypublishing.org

a decision to disregard the result was arrived at before its computation. Such occasions were very rare.

Of the 71 observations made, 7 are within 1 in a million of the mean, 14 are within 2, 28 within 5, 53 within 10, 66 within 15, and 70 within 20 in a million.

Only 1 determination out of the whole 71, and this one of the earliest, differs from the mean by so much as 1 part in 59,000.

The above facts constitute important evidence of constancy in both cell and balance. In fact, both current weigher and cell proved to be much more constant and trustworthy than the standard resistance, although the latter was very carefully made and annealed with a view to ensuring permanency.

Expressed in terms of the international ohm, as realised at the National Physical Laboratory, and of the ampere as given by the new current weigher, the value of C × R for the normal Weston cadmium cell is

$$1{\cdot}01830_5 \text{ at } 17^\circ \text{ C.}$$

This assumes that the value of g at Bushy is 981·19, a number probably correct to within 3 parts in 100,000. An uncertainty of this amount in g introduces a possible error of $1\frac{1}{2}$ parts in 100,000 in the value of the ampere, and, as all other probable errors are smaller in magnitude, it is important that a more accurate determination of g at Bushy should be made.

To realise the volt with an accuracy approaching that of the ampere as now known, it is necessary that an absolute determination of resistance of corresponding precision be undertaken. At the present time the uncertainty in the absolute value of the international ohm approximates to 4 parts in 10,000.

From the above value of C × R for the cadmium cell, together with the ratio of Clark to cadmium, viz.:—

$$\text{Clark at } 15^\circ \text{ C.} \div \text{cadmium at } 17^\circ \text{ C.} = 1{\cdot}406_6,$$

the E.M.F. of the Clark cell at 15° becomes

$$1{\cdot}432_3.$$

Two Appendices accompany the paper: one of these gives the numerical values of the constants in the series for calculating F and E, the complete elliptic integrals of the first and second kinds respectively; the other relates to the corrections in the forces between the coils of the balance arising from the finite thickness of the wire used, and from the fact that helices are substituted for current sheets. Neither of these corrections affects the results appreciably.

Some historical notes are given in the paper.